ECO2003F

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Costs

Chapter 8

- Costs in the short run
- Allocation production between 2 processes
- Relationship between MP, AP, MC and AVC.
- Costs in the long run
- Long run costs and the structure of industry
- Relationship between long run and short run cost curves
- Learning curve vs Economies of Scale

Please note, the chapter on cost curves is not the most pleasant, and is very long.

Your guide as to what to cover is:

Lecture slides
Tuts
tests

If I don't mention it in lectures, it is not important

EXCEPT

If it is revision from last year.

Bain's Kloof Pass

Built by convict labour in 1853 - 30km long
By Andrew Bain
No formal training (this should inspire you)

Initially laid with gravel

Where did they get the gravel from?
Why?
Couldn't they afford a gravel making machine?

Relative costs matter

Tying it all together

Ch 7 - how do we produce using inputs K, L

Now to consider input costs

i.e. **wages** (w), **capital costs** (r)

First we tackle **short run**, then **long run**

Short run - VC,FC,TC,MC,ATC,AFC,AVC
Long run - overall, how do we minimise costs?

Note: Lagrangians will ask:

Minimise a cost function,
s.t.
a production quota

Or

Maximise a production function,
s.t.
a cost constraint

The first one students usually get wrong in tests

Snow white and the 7 cost curves

Revision

- $FC = rK$
- $VC = wL$
- $TC = FC + VC$
- $AFC = FC/Q = rK/Q$
- $AVC = VC/Q = wL/Q$
- $ATC = TC/Q = (rK + wL)/Q$
- $MC = \Delta TC/\Delta Q$ or dTC/dQ

Be careful of units!

MC, AVC, AFC, ATC: Rand Cost per CD (Q),

Short run: **TC, VC**: Rand Costs per student (L)

FC: Rand cost

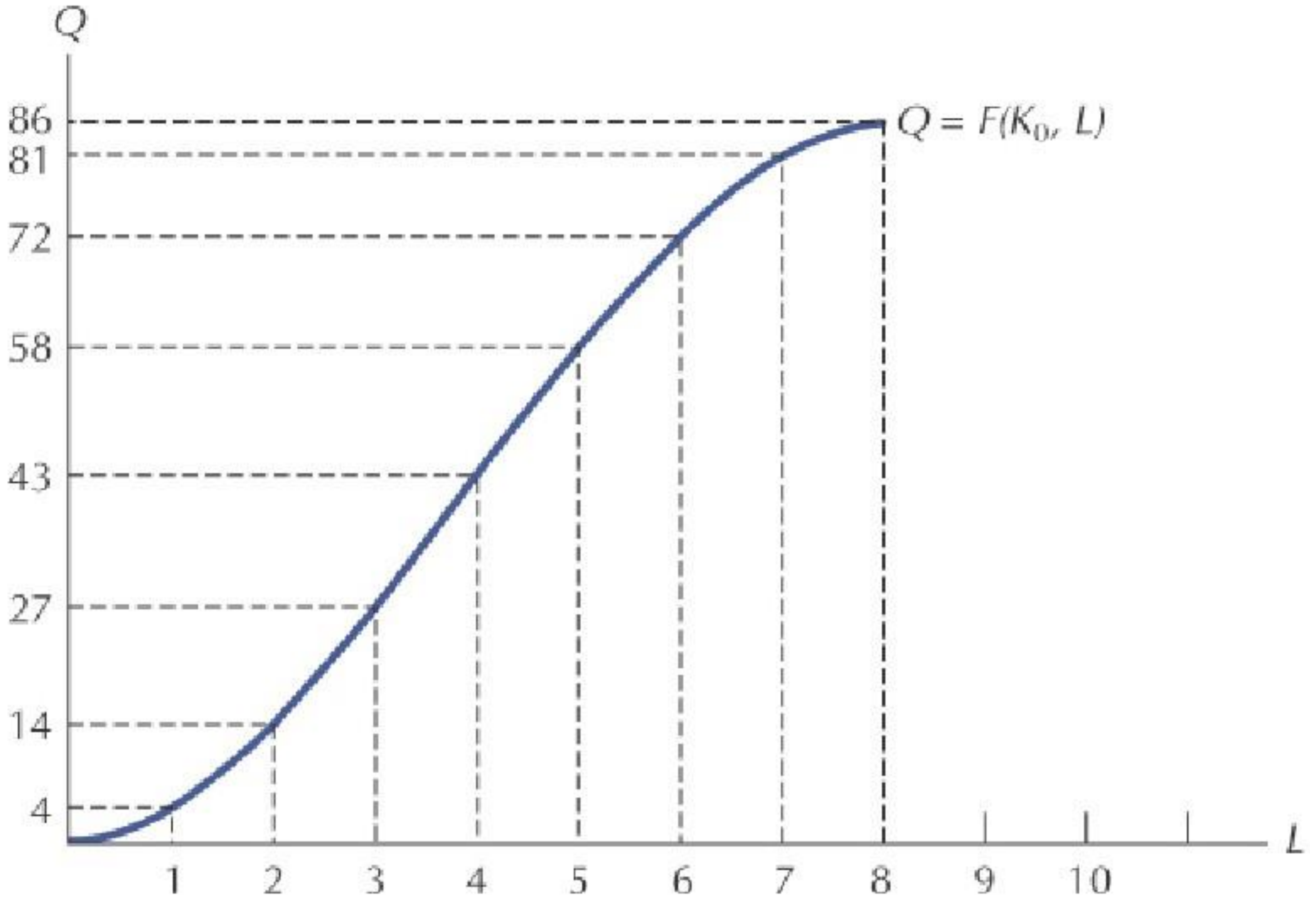
What is on the **axes** for a prodn fn? Cost fns?

What is VC dependent on? A function of? And FC?

What happens to VC as prodn increases?

Students (L)	CDs (Q)
0	0
1	4
2	14
3	27
4	43
5	58
6	72
7	81
8	86

Short Run Production: Q = F(L) **(Revision)**



We pay each student R100 (w)

The CD press costs R300 (r)

Output is measured in number of CDs (Q)

Output increases as we add 1 more unit of L

All figures for AVC, ATC, AFC, MC are R/unit

L	QFC	VC	TC	MC
0	0			
1	4			
2	14			
3	27			
4	43			
5	58			
6	72			
7	8			

L	Q	FC	VC	TC	MC
0	0	300	0	300	
1	4	300	100	400	100/4=25
2	14	300	200	500	10
3	27	300	300	600	7.69
4	43	300	400	700	6.25
5	58	300	500	800	6.67
6	72	300	600	900	7.14
7	8	300	700	1000	11.11

Total, Variable, and Fixed Cost Curves

(Revision)

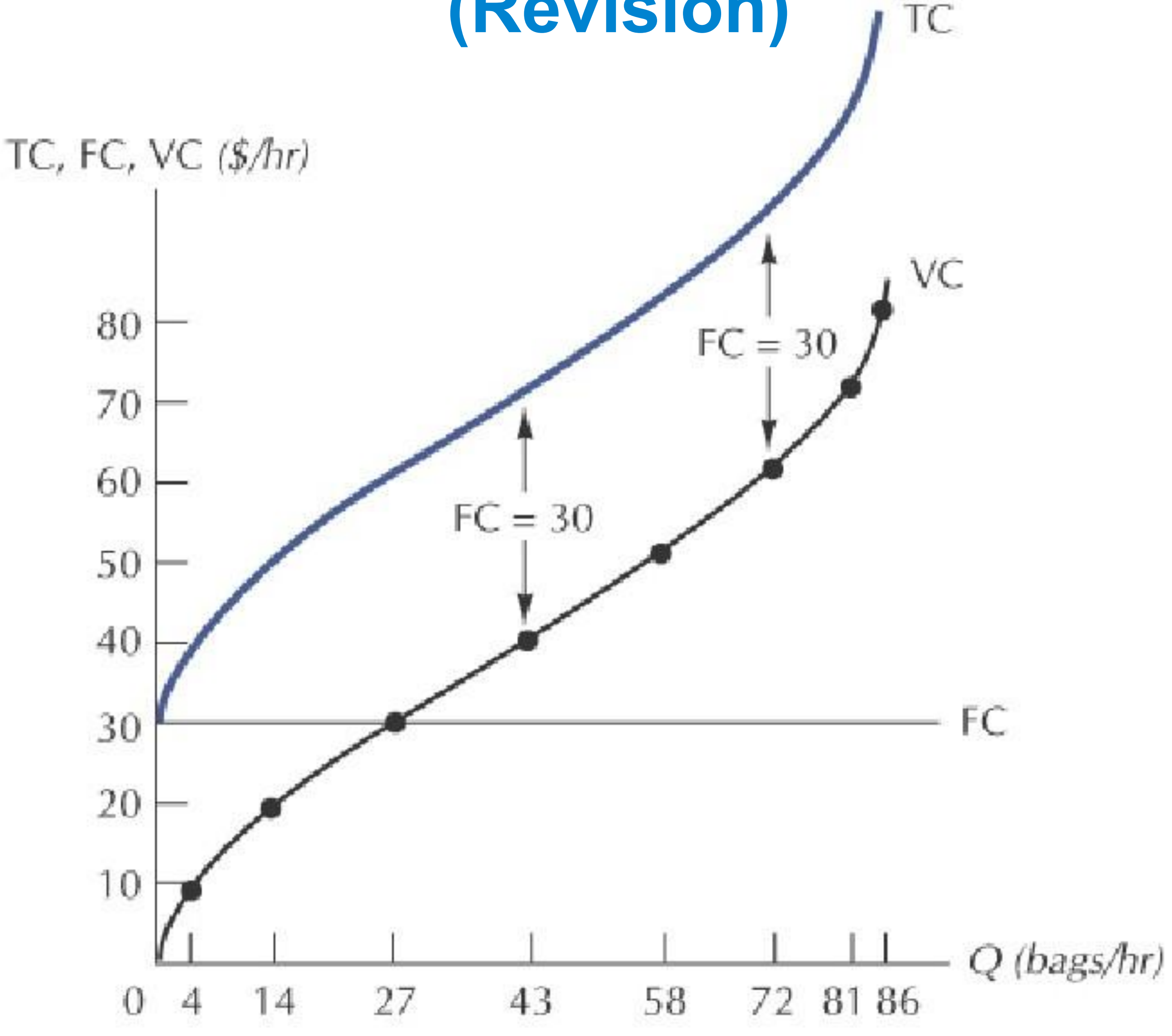
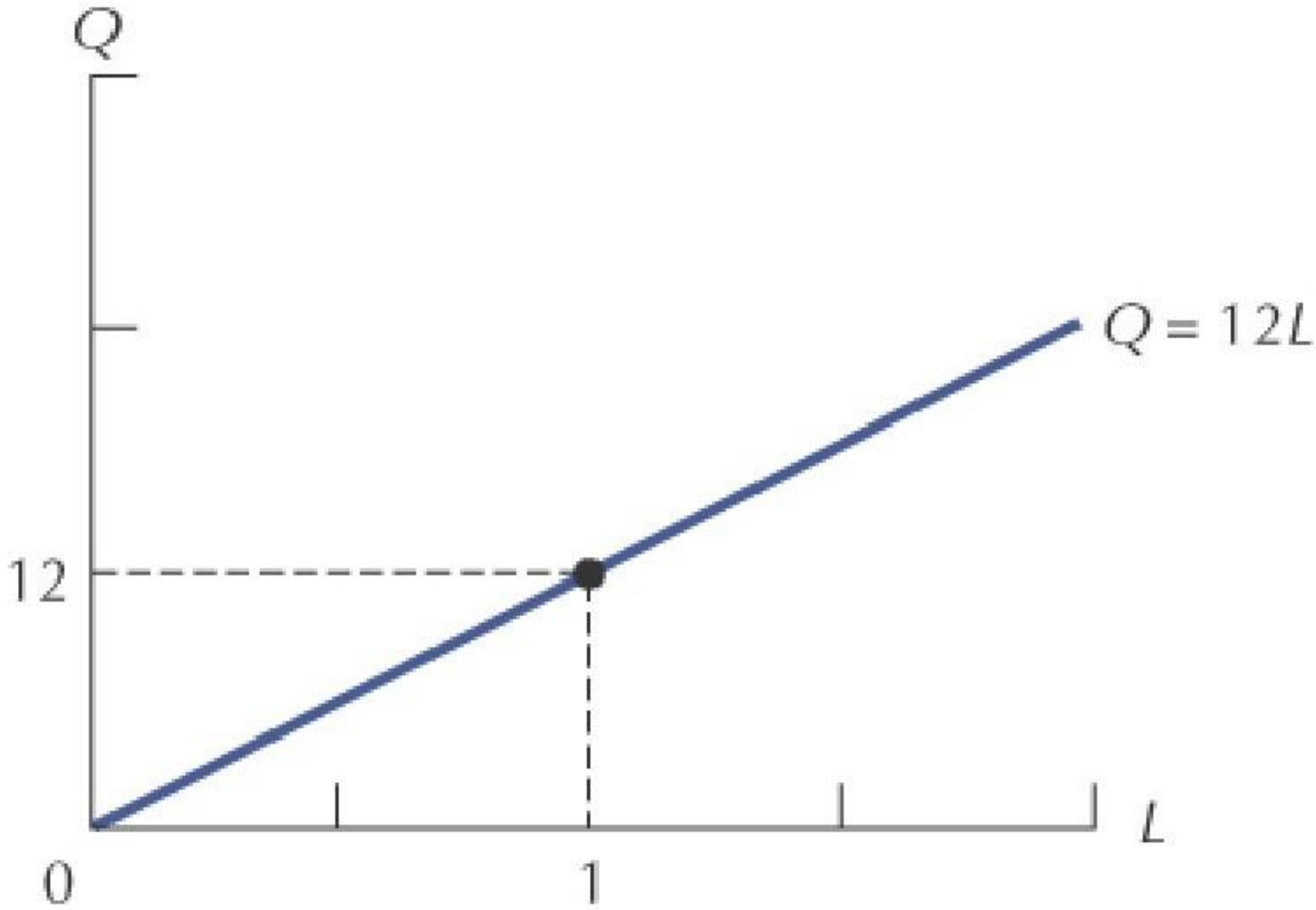


Figure 8.3: The Production Function $Q = 3KL$, with $K = 4$



Given $Q = 3KL$

Short run, K fixed at $K = 4$

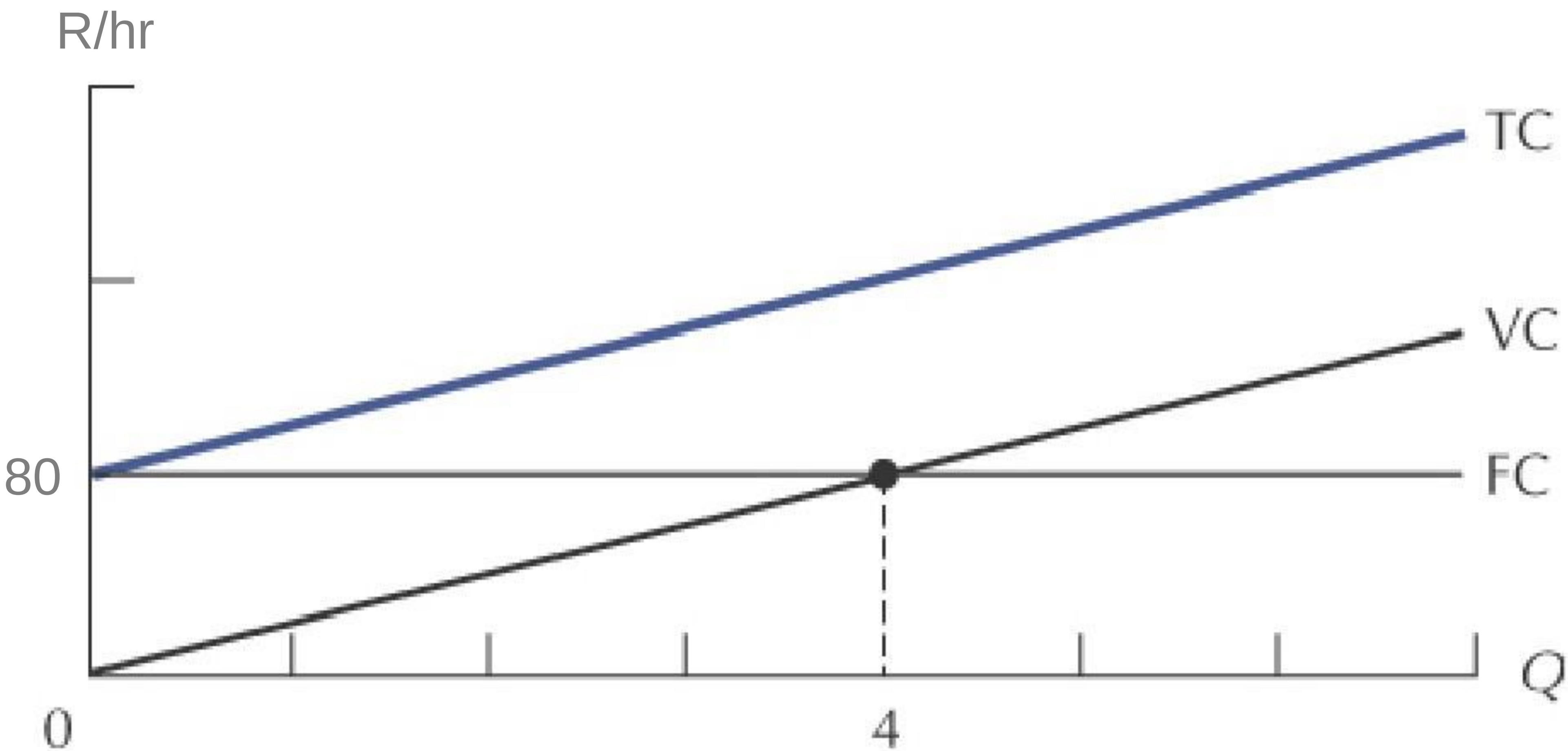
Costs - Given:
$w = 240$/hr
$r = 20$/machine hr

Work out:

SR fn: $Q = 3(4)L = 12L$ Therefore $L = Q/12$
$FC = rK = 80$/hr,
$VC = wL = 240L = 240(Q/12) = 20Q$

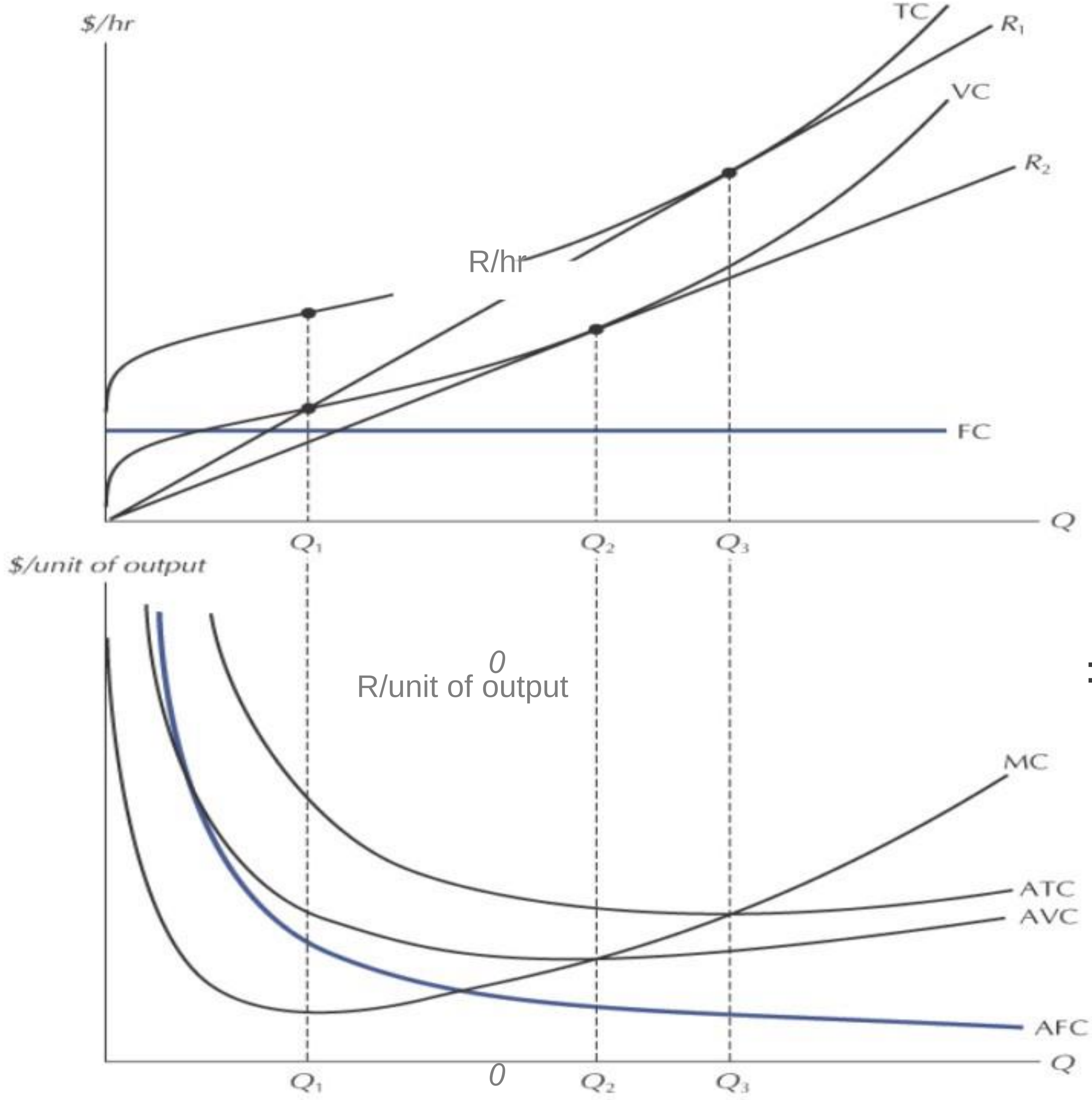
Why rewrite VC like this? What is TC?

Figure 8.4: The Total, Variable, and Fixed Cost Curves for the Production Function Q = 3*KL* **(Revision)**



Revision

- Where do costs increase at an increasing rate?
- What happens to FC as $Q \to \infty$? And AFC?
- What happens to VC as $Q \to \infty$? And AVC?
- What happens to FC, VC, AFC, AVC as $Q \to 0$?
- What is the distance between VC and TC?
- What is AVC geometrically? And AFC? ATC?
- How are ATC & AVC related as $Q \to \infty$?
- Where does MC intersect ATC and AVC?
- What happens to ATC & AVC if MC is below them?

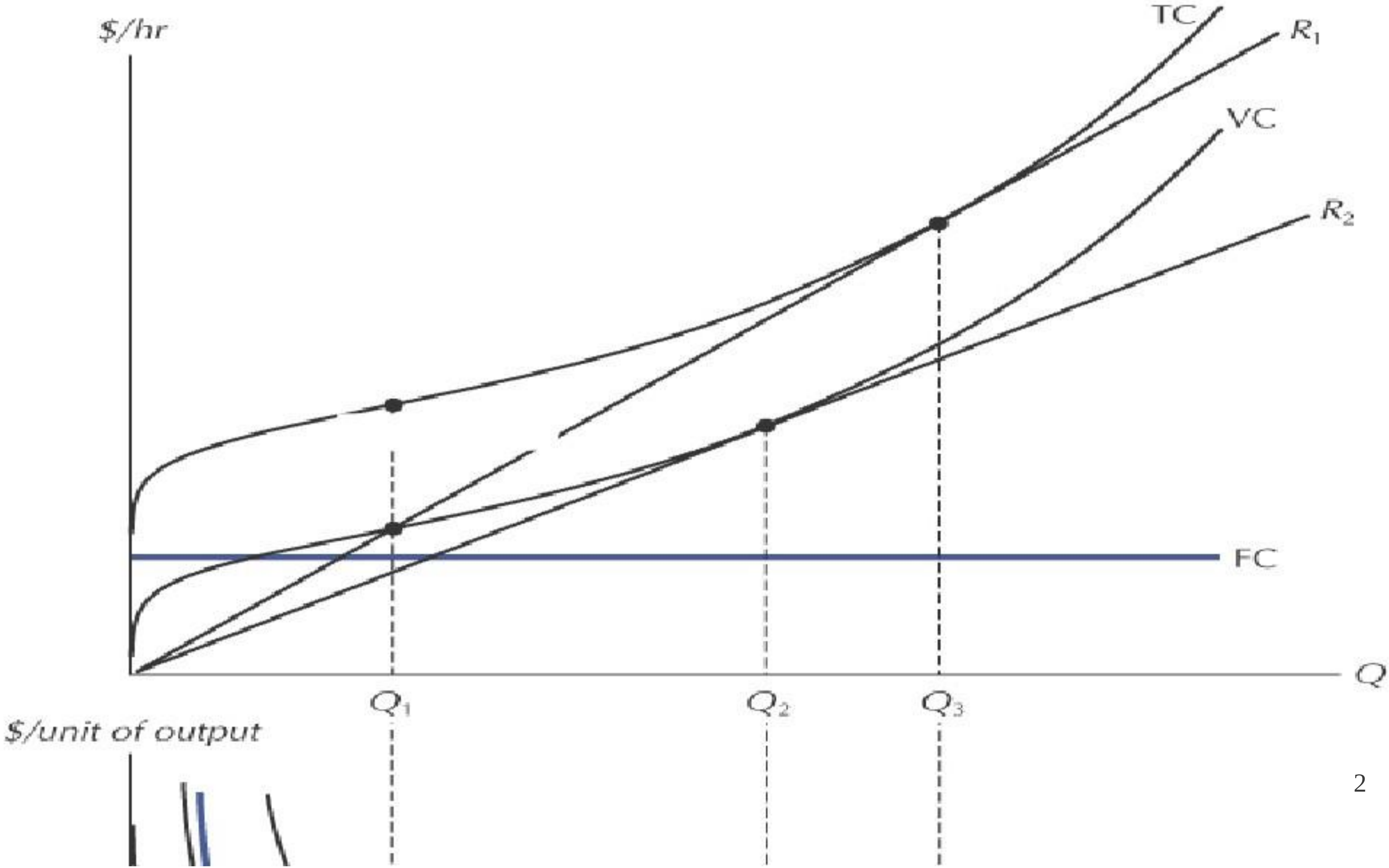


Revision

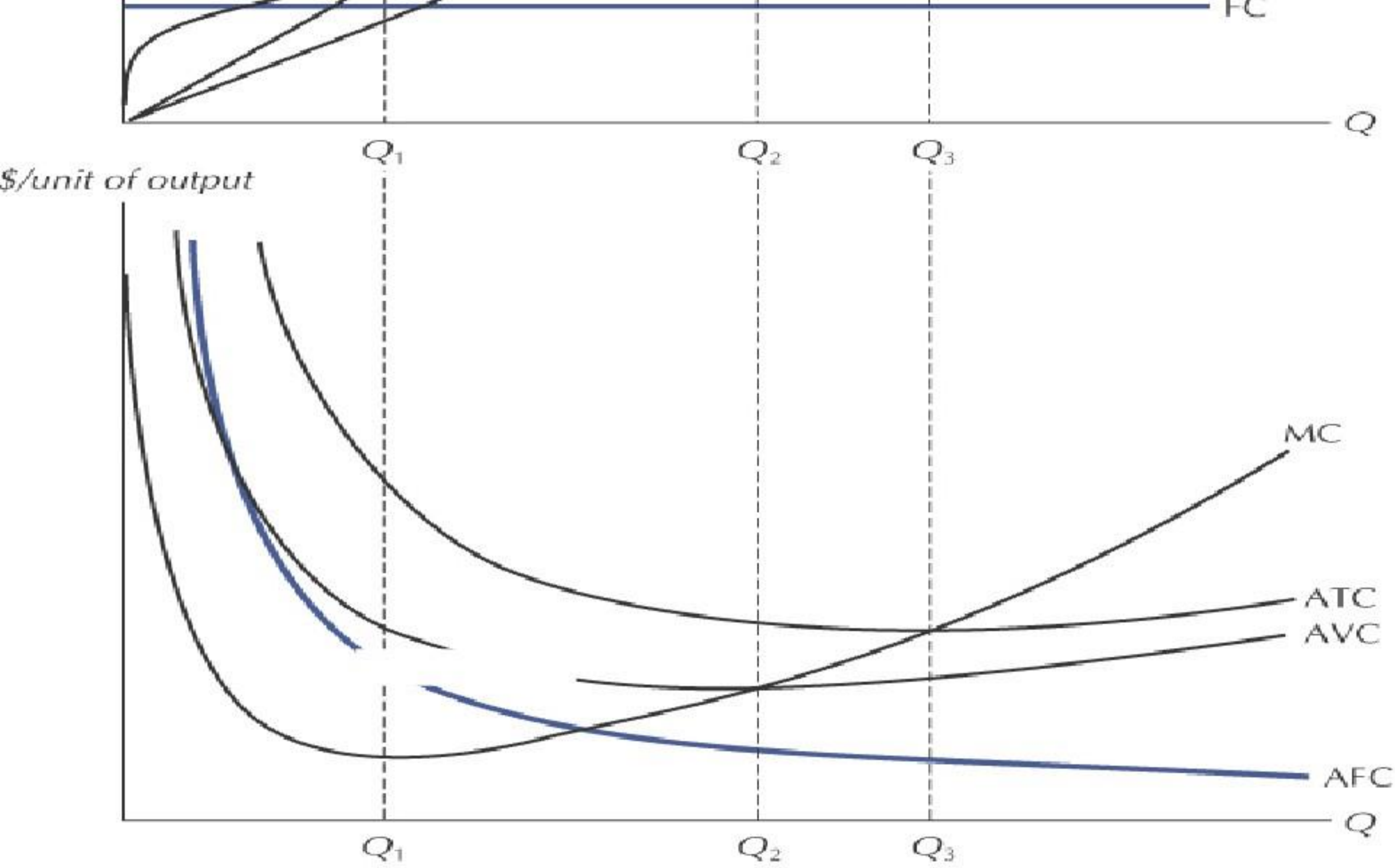
TC
= FC + VC

ATC
= AFC + AVC

AVC
= VC/Q



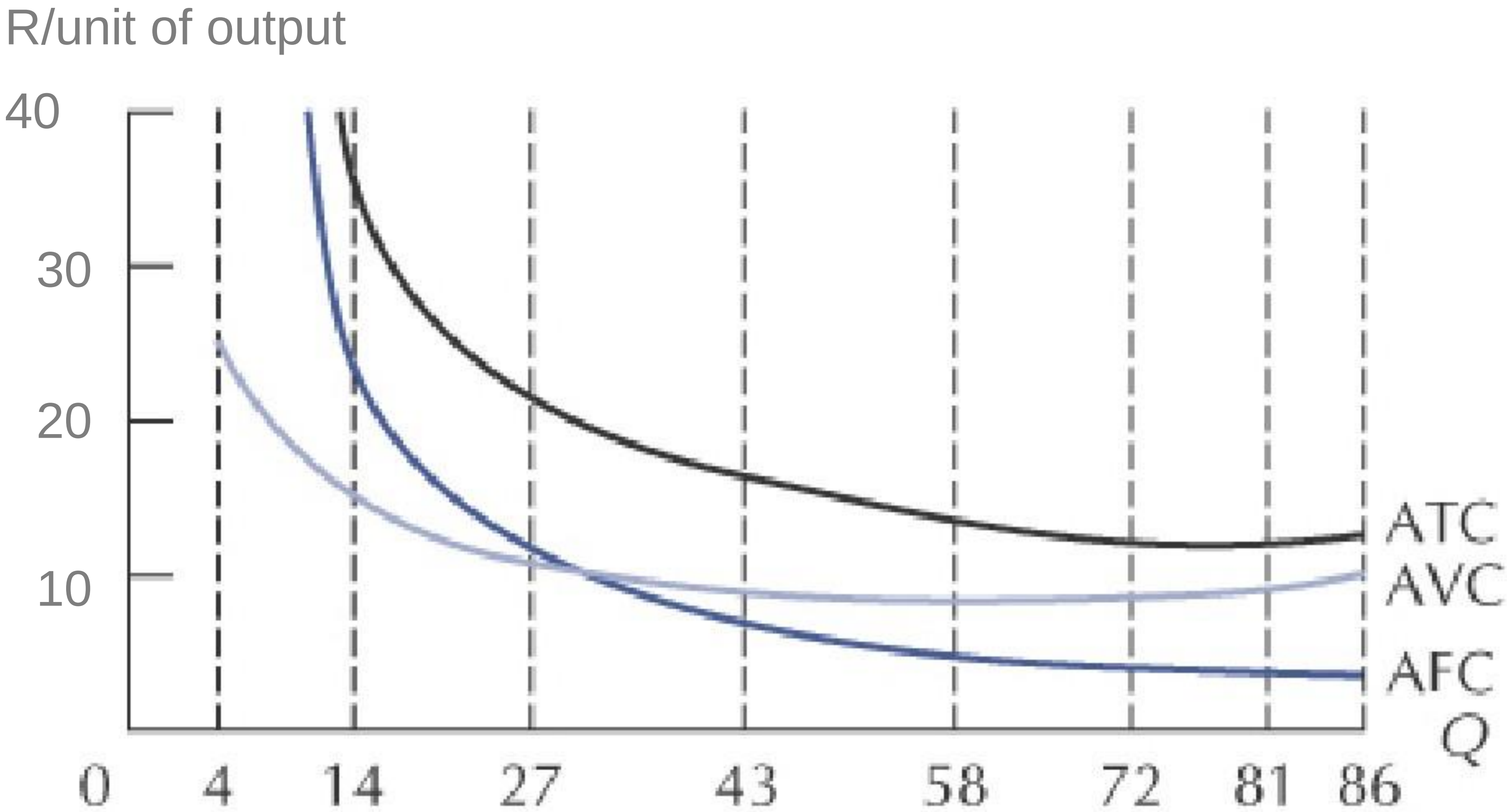
$/hr
TC
R_1
VC
R_2
FC
Q
Q_1
Q_2
Q_3
$/unit of output


FC
Q
Q_1
Q_2
Q_3
$/unit of output
MC
ATC
AVC
AFC
Q
Q_1
Q_2
Q_3

Q	FC	VC	TC	MC
0	300	0	300	
4	300	100	400	100/4=25
14	300	200	500	10
27	300	300	600	7.69
43	300	400	700	6.25
58	300	500	800	6.67
72	300	600	900	7.14
81	300	700	1000	11.11
86	300	800	1100	20

Q	AFC	AVC	ATC	MC
0	∞	-	∞	
4	300/4=75	100/4=25	400/4=100	100/4=25
14	21.43	14.29	35.71	10
27	11.11	11.11	22.22	7.69
43	6.98	9.30	16.28	6.25
58	5.17	8.62	13.79	6.67
72	4.17	8.33	12.50	7.14
81	3.70	8.64	12.34	11.11
86	3.49	9.30	12.79	20

Figure 8.6: Quantity vs. Average Costs
(Revision)



Revision: $Q = 3KL$

$r = 20$, $w = 240$, in the short run $K = 4$

Graph ATC, AVC, AFC, MC

$Q = 3KL = 3(4)L = 12L$, therefore $L = Q/12$

$TC = FC + VC$

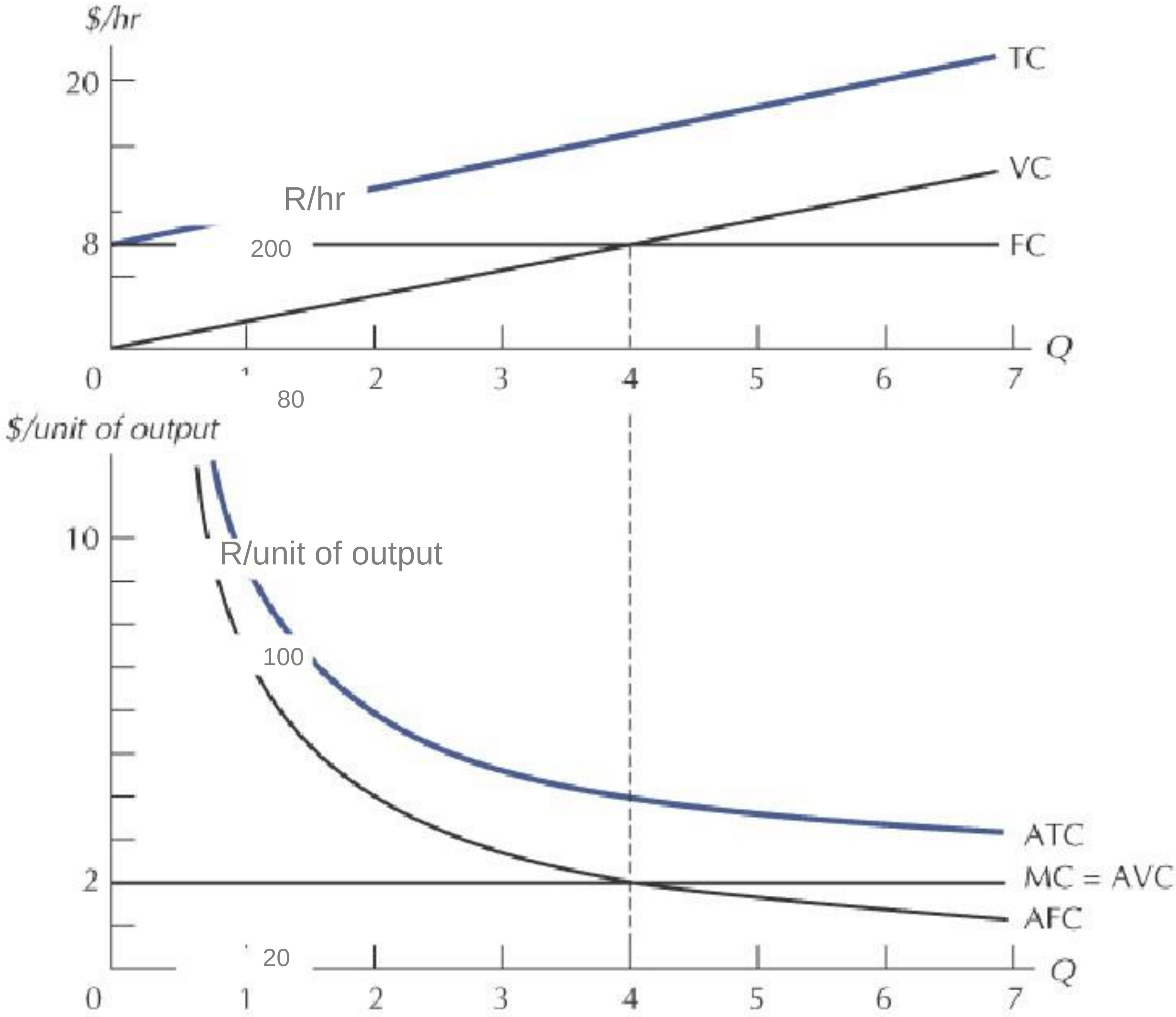
$TC = rK + wL$

$TC = 20(4) + 240(Q/12) = 80 + 20Q$

$FC = 80$, $VC = 20Q$

$ATC = TC/Q = 80/Q + 20$, $AVC = VC/Q = 20$

$AFC = 80/Q$, $MC = dTC/dQ = 20$



Revision

$Q=3KL$

More questions/notes

* Is MC = slope of TC = slope of VC?

* AFC declines over all ranges of Q, ATC and AVC both decline, and then start to increase

* At some pt, AVC will start to rise, but ATC will still fall for some time - why?
ATC = AVC + AFC
* Thus the increase in AVC will be offset by the decline in AFC for a while but not forever
* Which cost curve is the most important?
* MC intersects ATC,AVC at their minimum pts

When deciding how much to produce:

- produce until MP = 0
- or if there are 2 processes, produce until MP is the same in each process (even though APs might be different)
- now, produce until MCs are the same in both processes (even though ACs may be different)

Why?

If the MPs, or the MCs aren't equalised, then we could do better

Allocating Prodn with Processes A,B

(Revision)

Given the following cost curves:

$MC_A = 12Q_A$, $ATC_A = 16/Q_A + 6Q_A$

$MC_B = 4Q_B$, $ATC_B = 240/Q_B + 2Q_B$

$Q = Q_A + Q_B$. Why?

What is the least costly way to produce $Q=32$?

Set $MC_A = MC_B$. Why?

$12Q_A = 4Q_B$ and we can substitute $Q_B = 32 - Q_A$

$12Q_A = 4(32 - Q_A) \rightarrow Q_A = 8$, thus $Q_B = 24$. Why?

$MC_A = 12Q_A$, $ATC_A = 16/Q_A + 6Q_A$

$MC_B = 4Q_B$, $ATC_B = 240/Q_B + 2Q_B$

$Q_A = 8$, $Q_B = 24$.

$MC_A = MC_B = 96$. Why?

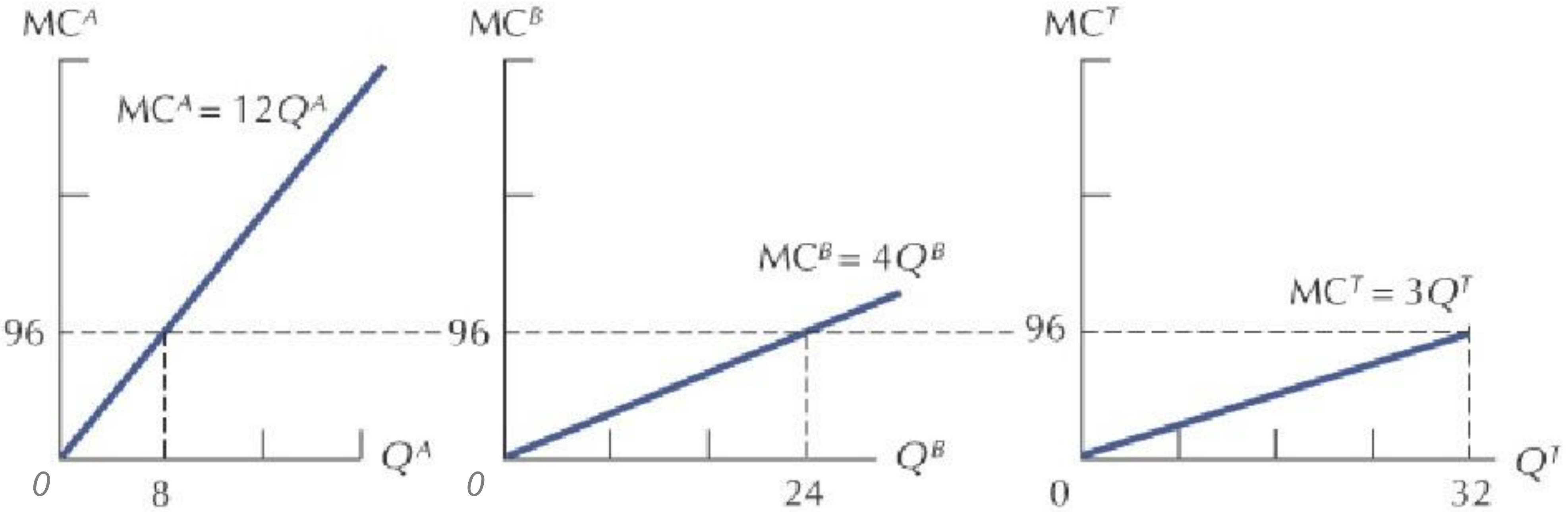
$TC_A = 16 + 6Q_{2A}$ $TC_B = 240 + 2Q_{2B}$

Overall $TC = 16 + 6Q_{2A} + 240 + 2Q_{2B}$

ATC_A = R50/unit, ATC_B = R58/unit

Shouldn't they be equal? What about TC_A & TC_B?

Figure 8.8: The Minimum Cost Production Allocation **(Revision)**



What is the **relationship between MC and MP**?

We know MC = ΔTC/ΔQ = ΔVC/ΔQ = ΔwL/ΔQ

If L is the only variable factor

If we pay all workers the same

MC = wΔL/ΔQ (taking the w out)

But ΔL/ΔQ = 1/MP

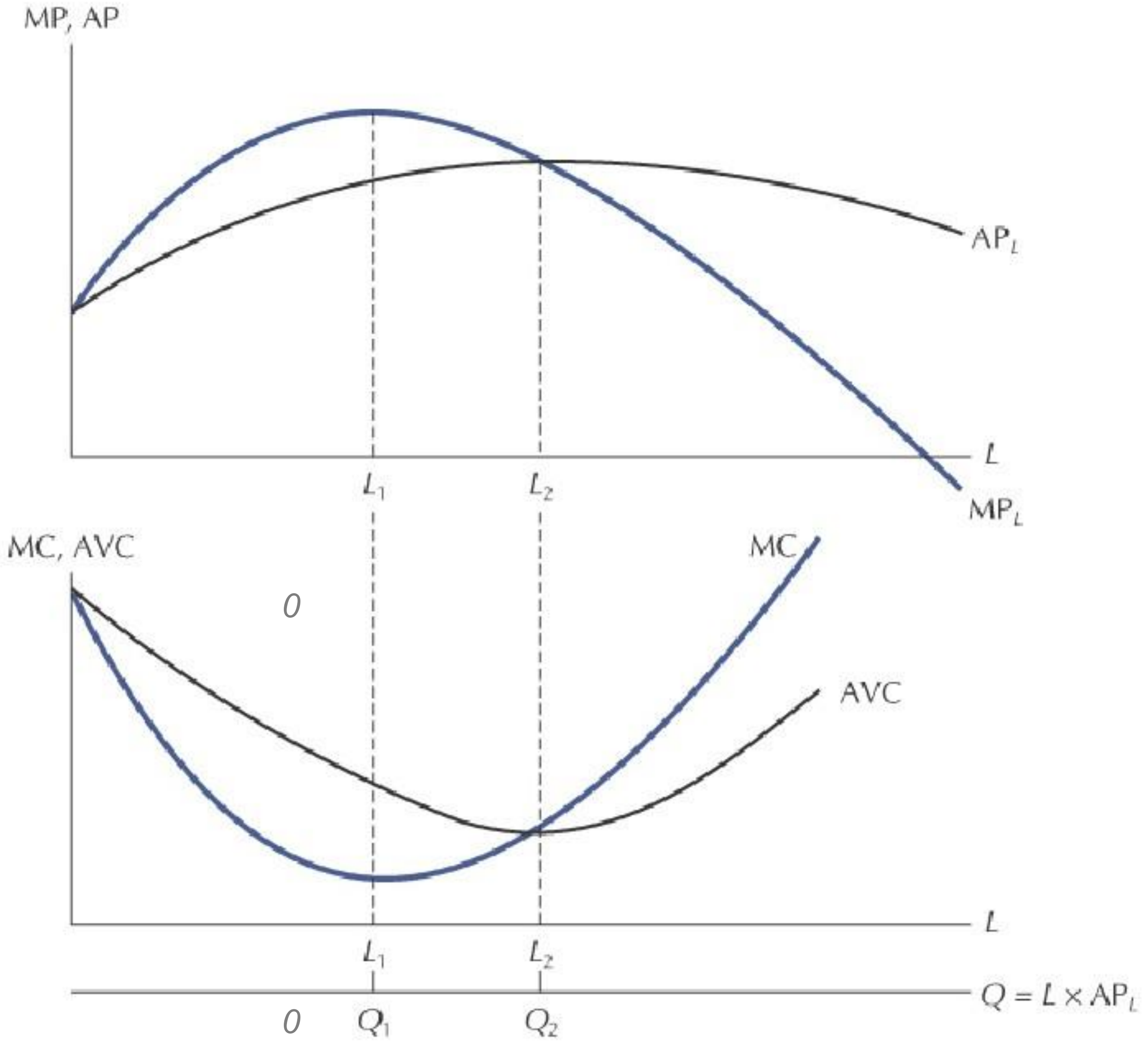
Therefore MC = w/MP

Similarly (check)

AVC = w/AP (p272)

So as w goes up, or MP goes down, MC goes up

MP, AP, MC, & AVC
(Revision)



So **MC** is minimised when **MP** is maximised

And **AVC** is minimised when **AP** is maximised

How do I work out Q if I know L?

Note the graphs the **labels on the axes**!

Now we turn to the **long run** (K,L variable)

Both can change, we now consider both in the cost function

Now things start to get a bit more realistic:

We know $C = rK + wL$

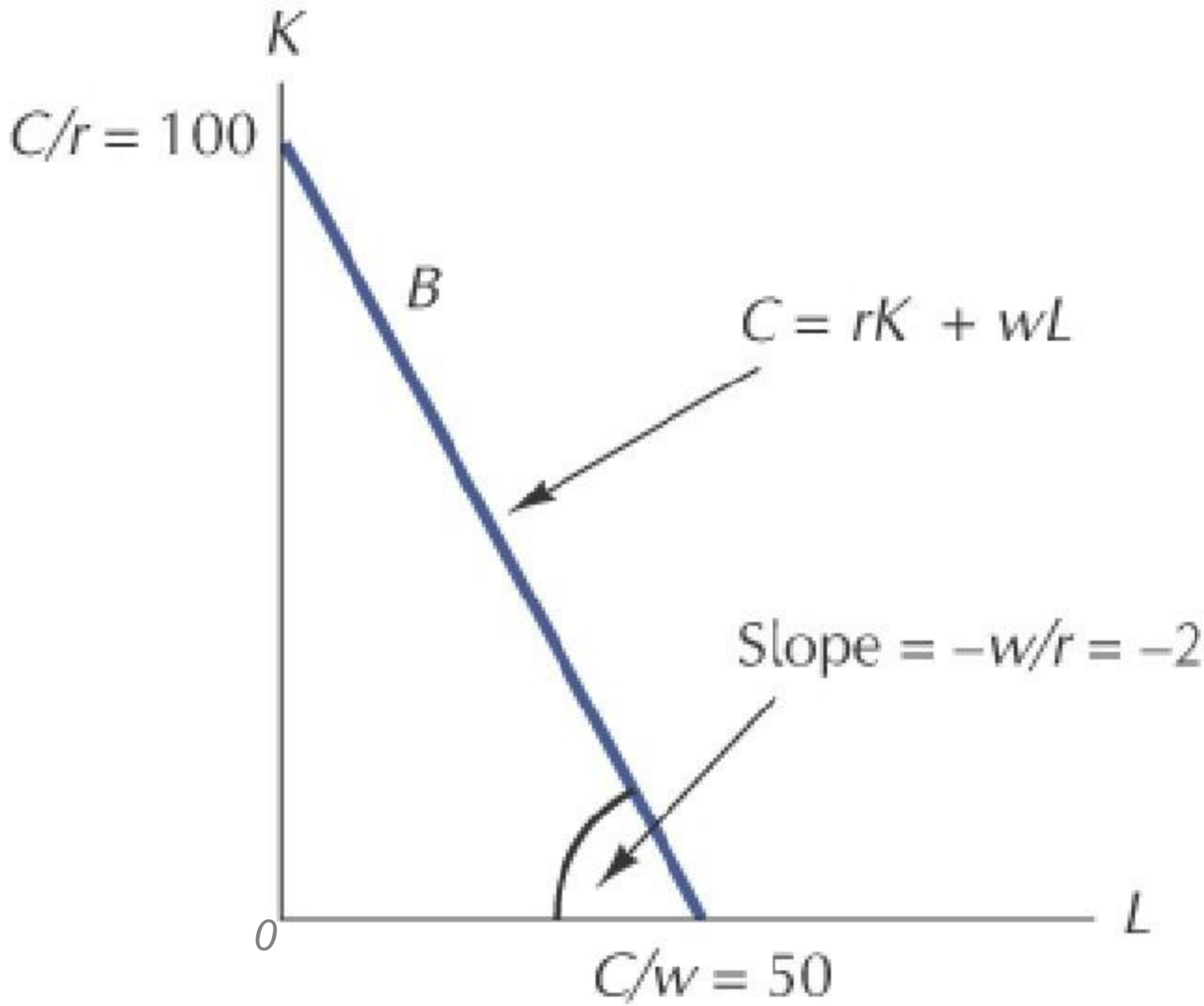
C is our budget amount we can spend on K,L

C/r = amt of K, if only spend C on K
C/w = amt of L, if only spend C on L

Solve for K: $\mathbf{K = -wL/r + C/r}$

Given $r = 20$, $w = 40$, $C = 20K + 40L$
We can solve for K and graph.

Figure 8.10: The Isocost Line



Now we can ask these questions:

How do we minimise costs, given an output quota

Or

How do we maximise output, subject to a cost constraint

We will end up with:

$$MP_L/MP_K = w/r$$

Why? First, let's look at the graphs.

Figure 8.11: The Maximum Output for a Given Expenditure

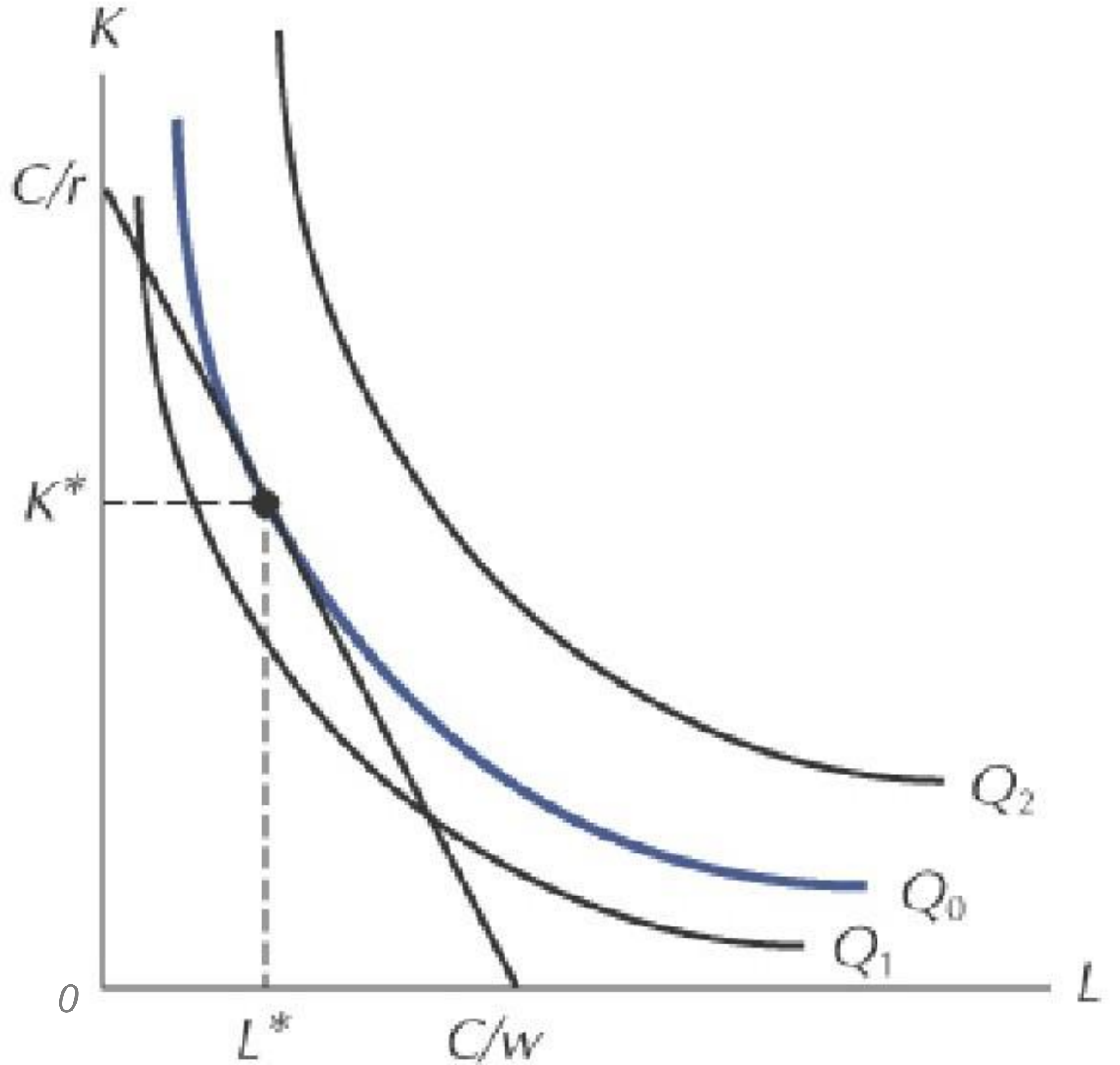
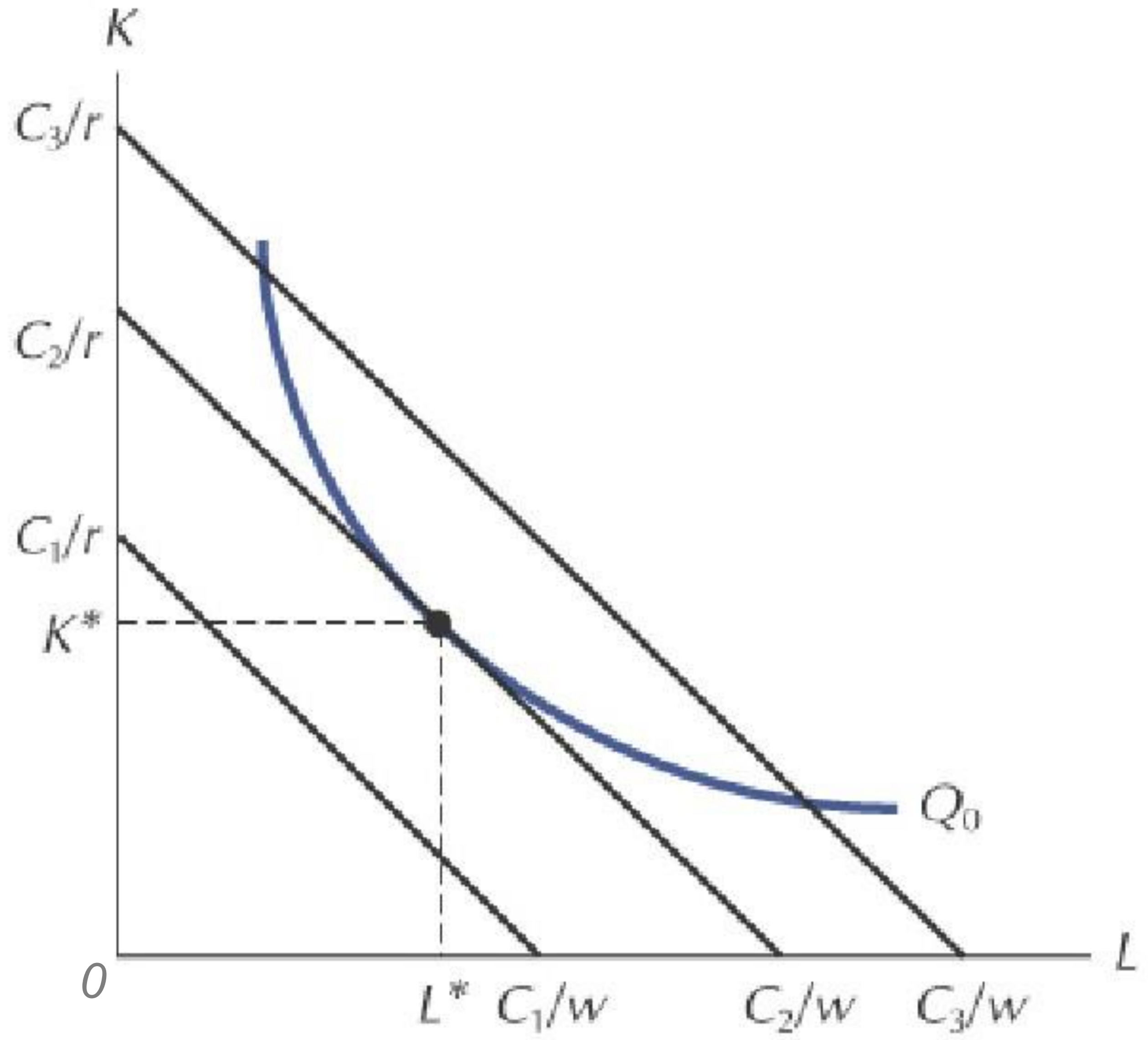


Figure 8.12: The Minimum Cost for a Given Level of Output



The **pt of tangency** between isoquant and isocost.

Slope isoquant = MP_L/MP_K

Slope isocost = w/r

So:
$\mathbf{MP_L/MP_K = w/r}$

We also know that at the optimum pt:
$\mathbf{MP_L/w = MP_K/r}$

The extra output from an extra rand spent on each input is the same for each input

$MP_L/w \neq MP_K/r$

i.e. 30 units per rand on K>20 units per rand on L,

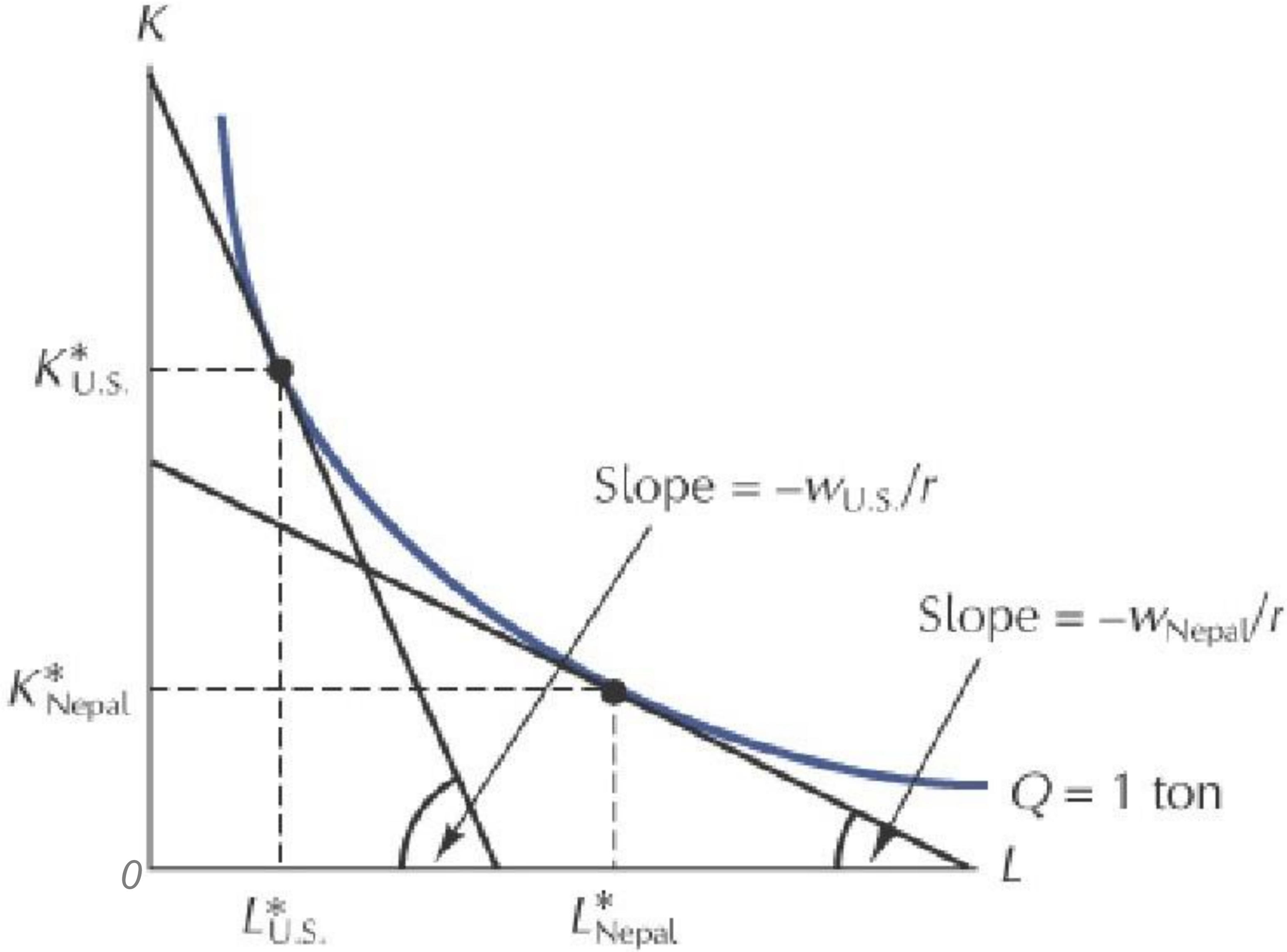
Then we ought to divert resources to more K, and less L

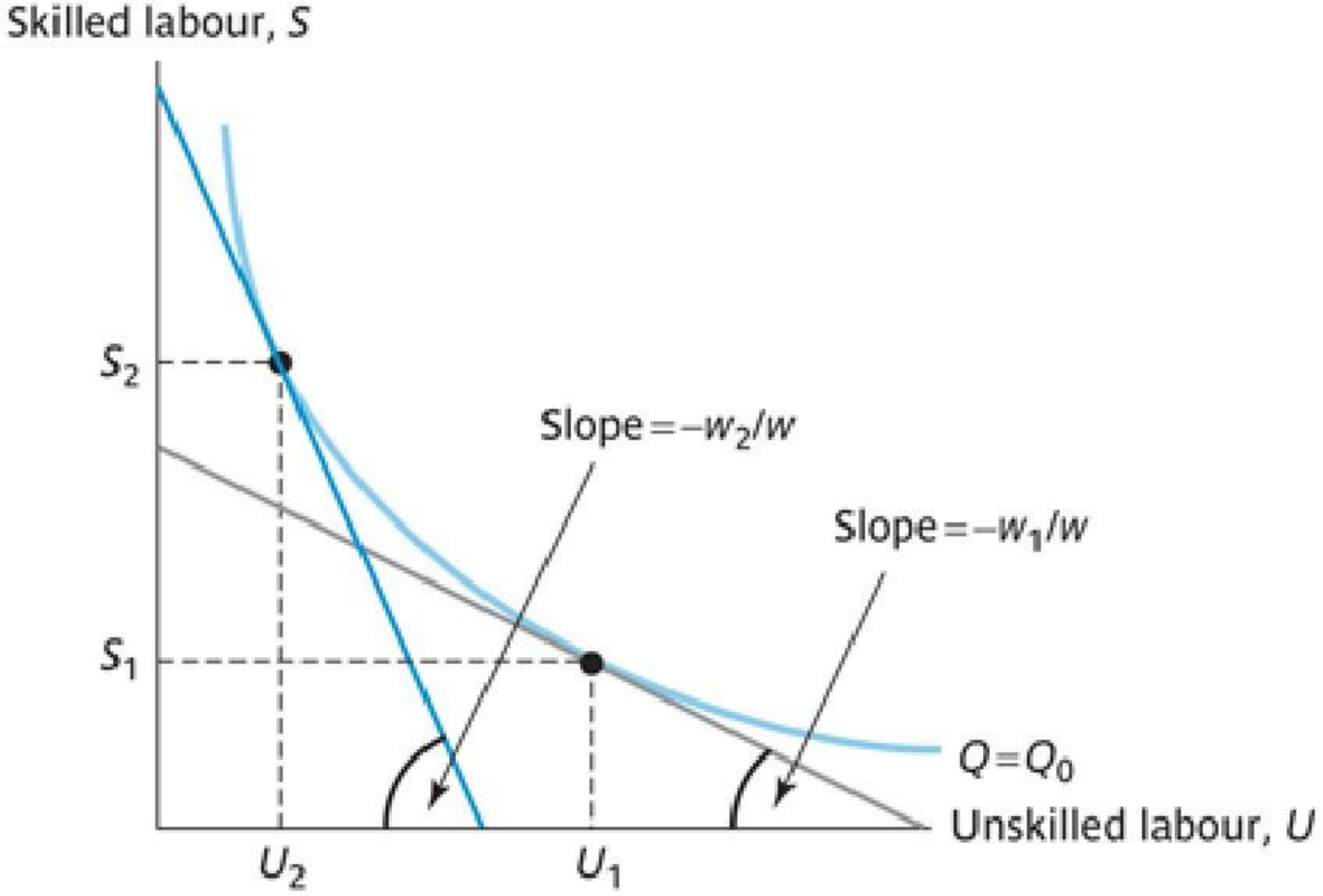
At the point of minimum costs, the ratio of MP to P is the same for each input.

The next slides should help us figure out Andrew Bain, and also why unions like minimum wages.

P278, price skilled = w, minimum wage pushes wages of unskilled from w1 to w2

Figure 8.13: Different Ways of Producing 1 Ton of Gravel





Skilled labour, S
S2
S1
Slope = –w2/w
Slope = –w1/w
Q = Q0
Unskilled labour, U
U2
U1

Unions shouldn't care about minimum wages - their members tend to be skilled

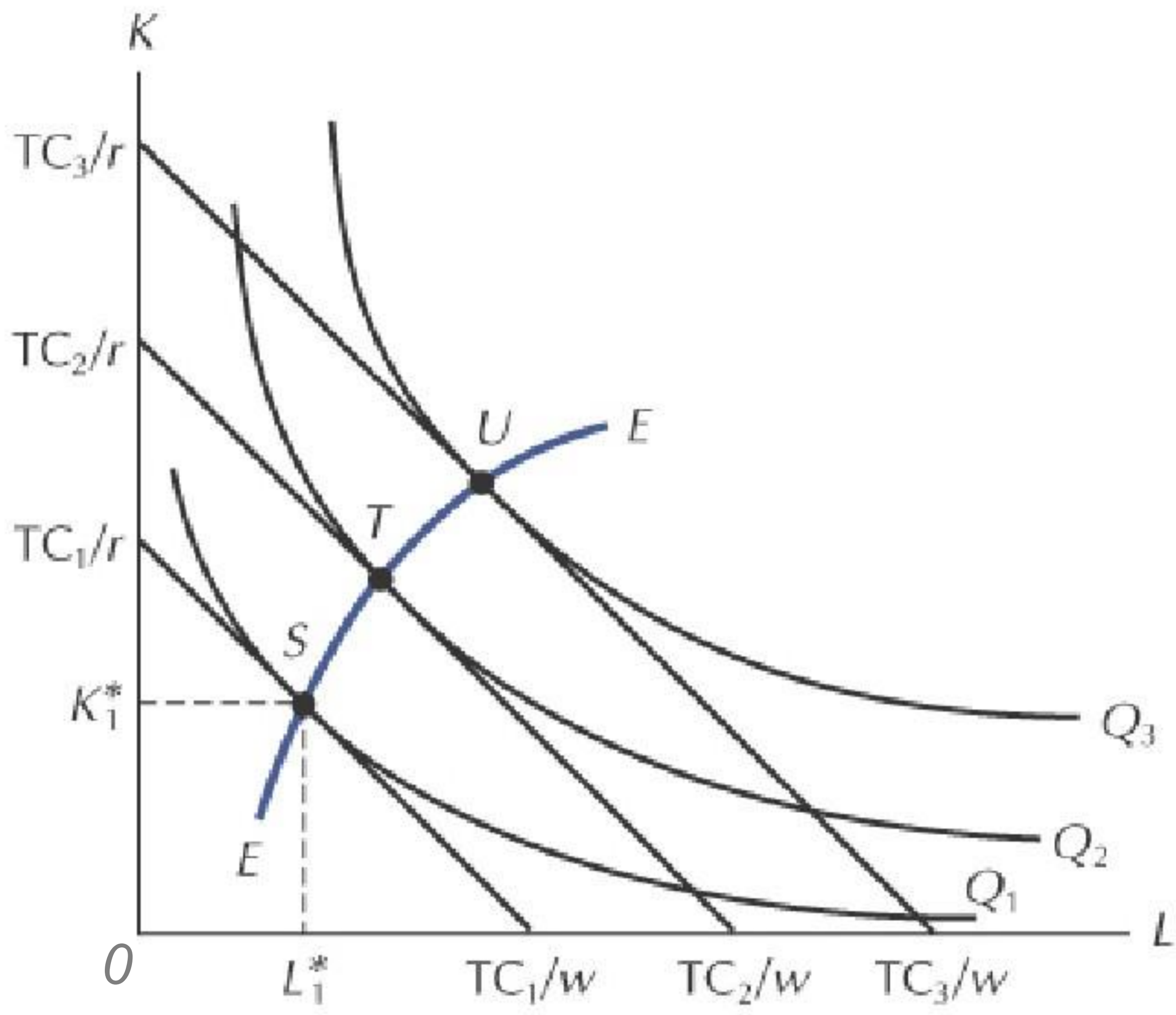
If a minimum wage becomes law
The price of unskilled labour rises

The ratio of unskilled to skilled wages rises

We're now on a different isocost - with more skilled, and less unskilled

The minimum wage law has “inadvertently” pushed up the demand for skilled labour

Figure 8.15: The Long-Run Expansion Path



With fixed r and w, we can see the least costly way to produce each level of output along line E (which joins all the pts of tangency)

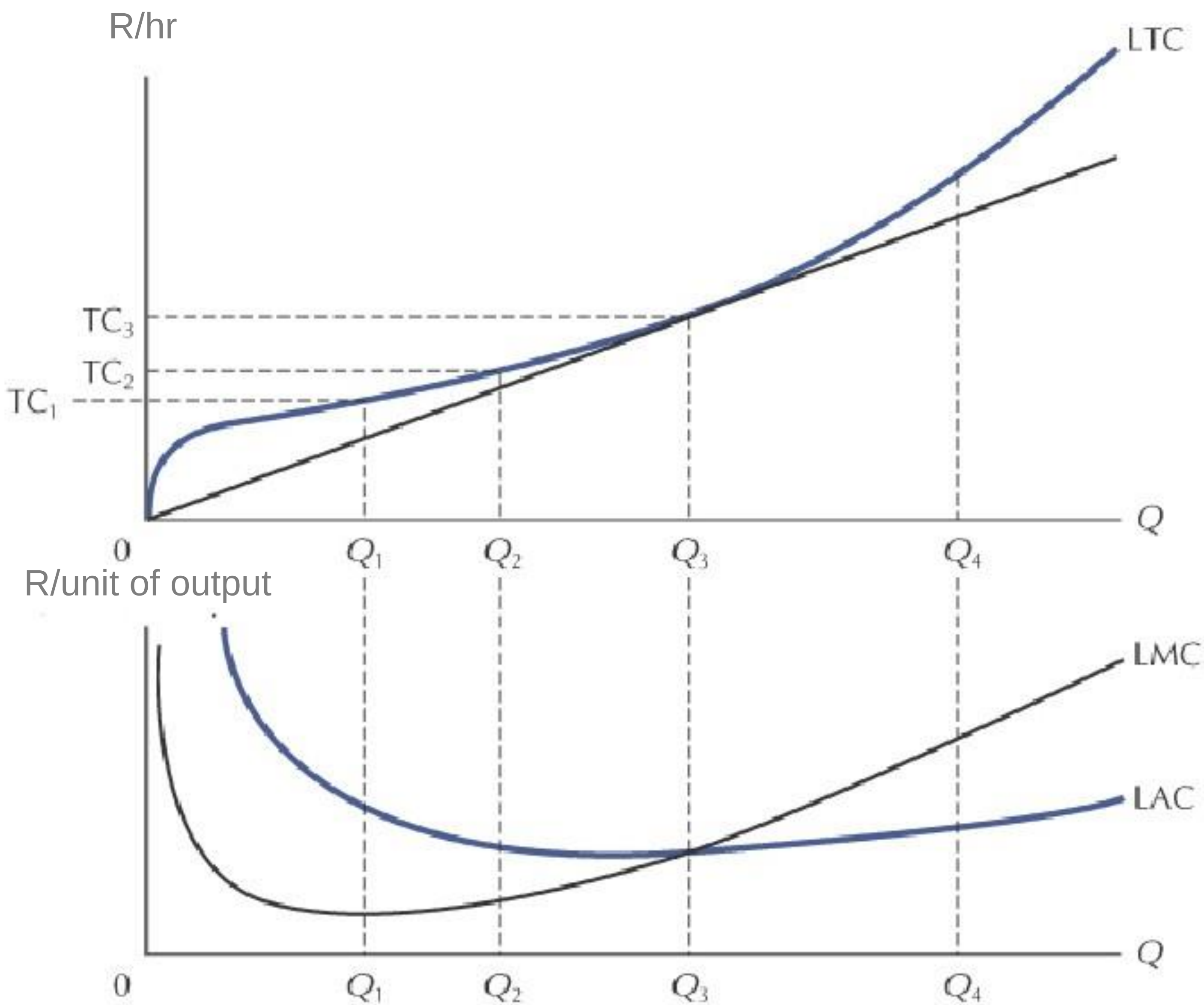
How do we now get LR TC?

From the expansion path, we can read off each level of Q and C, and plot them.

NB For long run costs:
All inputs are variable

Where did all the curves go? With no fixed costs, the picture is a lot simpler now.

Figure 8.16: The Long-Run Total, Average, and Marginal Cost Curves



Why does TC pass through the origin?

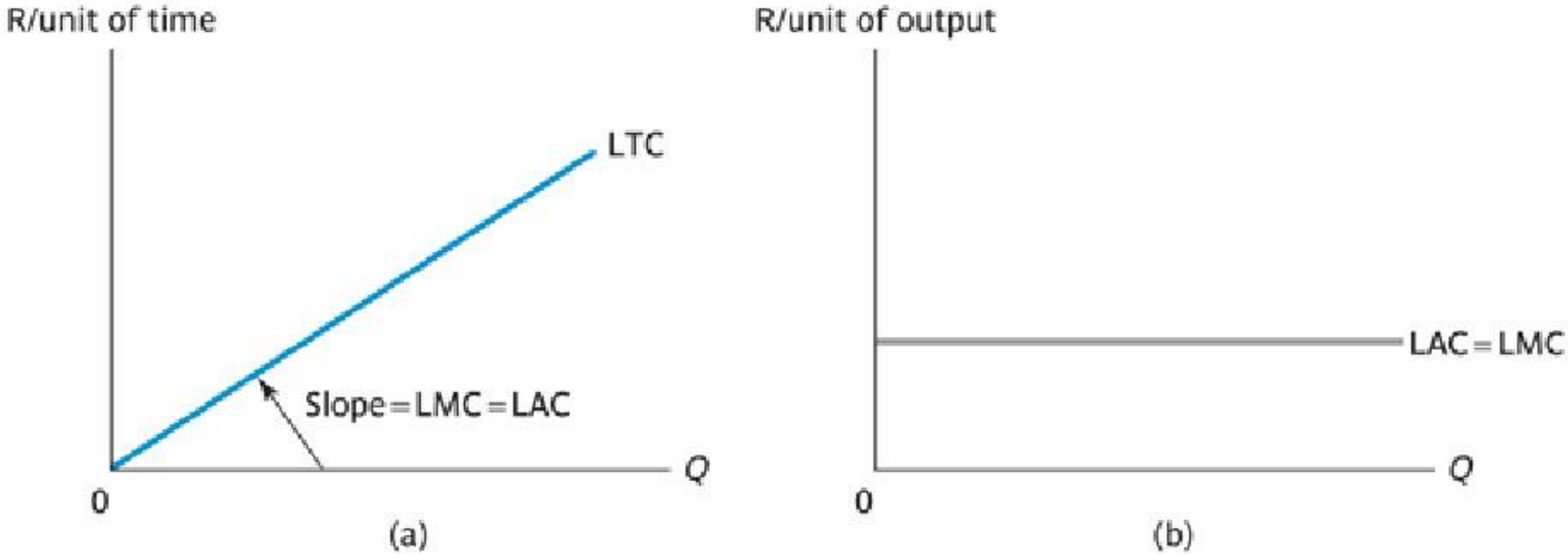
All the usual relationships hold, as they did for SR

Note, we do **not cover** from
P282 until P289, first paragraph

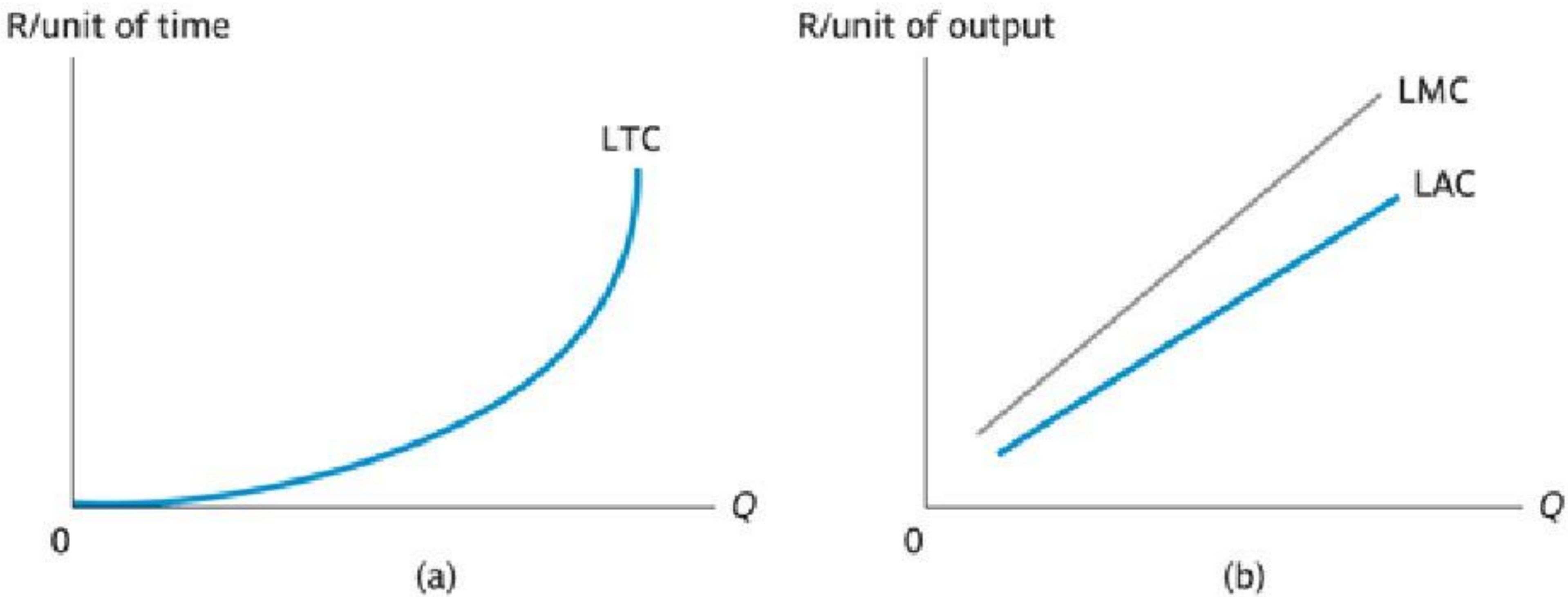
We **do cover** the Lagrangian box on P288

What about returns to scale?

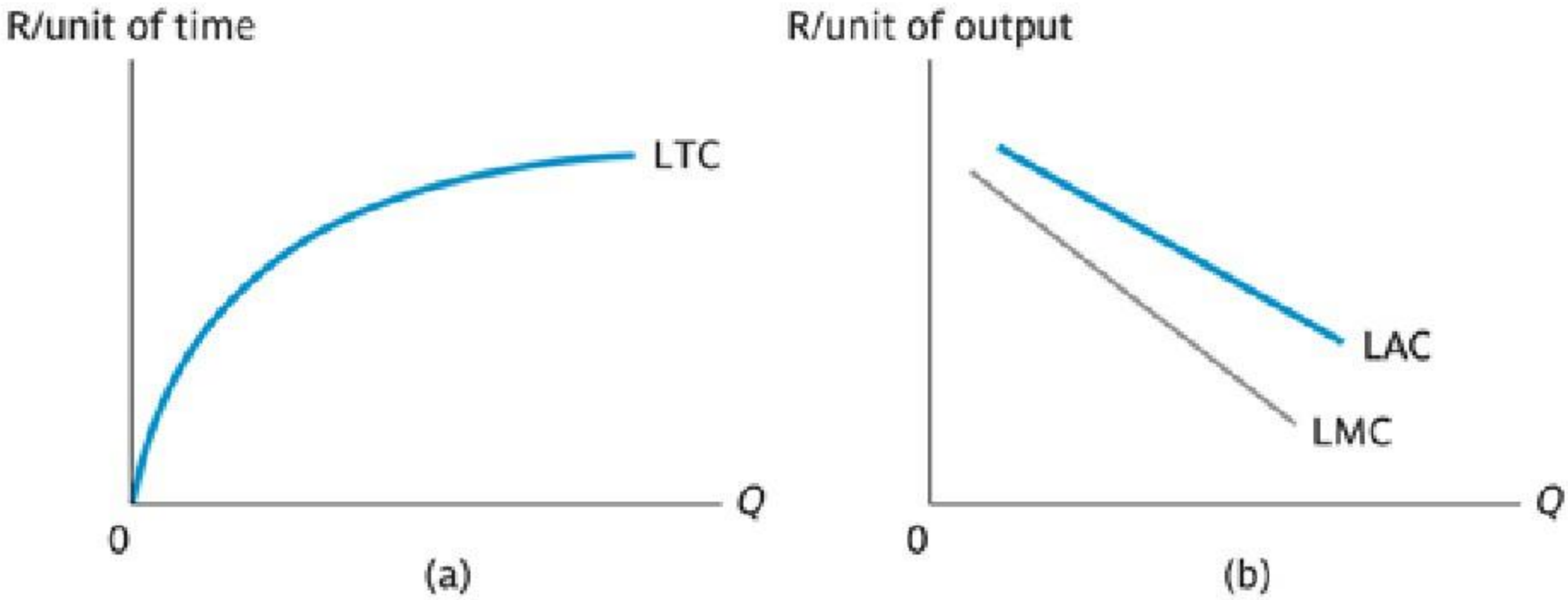
Constant Returns



Decreasing Returns



Increasing Returns



Minimise Costs subject to an Output Constraint

Minimise $(rK + wL)$ s.t. $F(K,L) = Q_0$

$$\mathcal{L} = rK + wL + \lambda(Q_0 - F(K,L))$$

$\mathcal{L}_L = w + \lambda(-F_L) = 0$

$\mathcal{L}_K = r + \lambda(-F_K) = 0$

$\mathcal{L}_\lambda = Q_0 - F(K,L) = 0$

$w/r = F_L/F_K = MP_L/MP_K$

$F_L/w = F_K/r$ i.e. $MP_L/w = MP_K/r$

Given $Q = F(K,L) = K^{1/2}L^{1/2}$

$P_K = 4$, $P_L = 2$, find the values of K and L that minimise the cost of producing 2 units of output

$$TC = 4K + 2L \text{ s.t } Q = K^{1/2}L^{1/2} = 2$$

$$L = 4K + 2L + \lambda(2 - K^{1/2}L^{1/2})$$

$L_K = 4 - \lambda(1/2K^{-1/2}L^{1/2}) = 0$

$L_L = 2 - \lambda(1/2K^{1/2}L^{-1/2}) = 0$

$L_\lambda = 2 - K^{1/2}L^{1/2} = 0$

$2 = L/K$ Therefore $L = 2K$. Sub into L_λ

$$TC = 4K + 2L \text{ s.t } Q = K^{1/2}L^{1/2} = 2$$

$L_K = 4 - \lambda(1/2K^{-1/2}L^{1/2}) = 0$

$L_L = 2 - \lambda(1/2K^{1/2}L^{-1/2}) = 0$

$L_\lambda = 2 - K^{1/2}L^{1/2} = 0$

$2 = K^{1/2}(2K)^{1/2}$

$2 = 2^{1/2}K$ Therefore $K = 2^{1/2}$ and $L = 2(2^{1/2}) = 2^{3/2}$

We can now find the minimum value for costs:

$TC = 4(2^{1/2}) + 2(2^{3/2}) = 2^{7/2}$ (check this)

Learning Curve vs Economies of Scale

As firms **learn** more about the production process, **average production cost can drop** (for ALL levels of output) (shift of the curve)

This effect peters out after a while

Economies of scale - when we increase production and see average costs drop (move along curve)

New firms more likely to see a **learning effect**

Figure 8.26 The Learning Curve

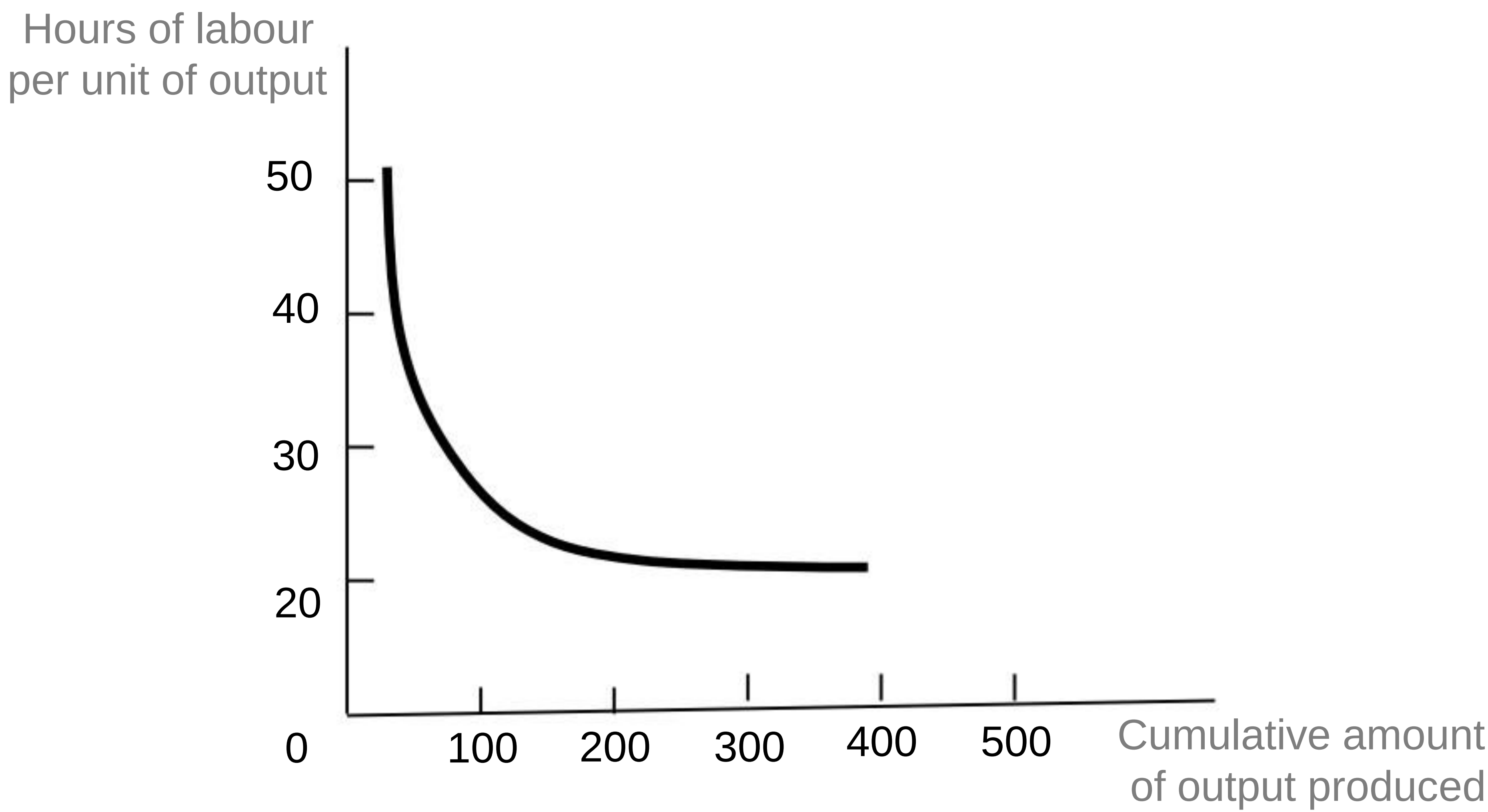


Figure 8.27: Economies of scale versus learning effect